Holdup and Pressure Drop for Two-Phase Slug Flow in Inclined Pipelines

R. H. BONNECAZE, W. ERSKINE, JR., and E. J. GRESKOVICH

Esso Mathematics and Systems, Inc., Florham Park, New Jersey 07932

The simultaneous flow of gas and liquid in pipes is frequently encountered in industry. Consequently, considerable effort has been expended in developing reliable techniques for calculating holdup and pressure drop for gas-liquid flow. Most of the recent work has concentrated on either vertical or horizontal flow. However, in some industrial environments, notably oil- and gas-gathering systems, flow is neither vertical nor horizontal.

In this paper we present some results from a recent study of gas-liquid flow in pipelines inclined ± 10 deg. from the horizontal. We shall concentrate our attention on the slug-flow regime because we found this regime to predominate in uphill and horizontal flows for conditions typically encountered in pipeline applications. Stratified flow tends to dominate in the downhill situation but even for this pipeline orientation, slug flow can exist if the flow rate is sufficiently large.

At the outset of this study it was decided that a fundamental approach would probably have the greatest potential for success. Therefore an extensive review was made of the two-phase literature in order to determine (1) the flow regimes encountered in two-phase flow and (2) the types of models proposed for these regimes. On the basis of this information it was possible to develop the model for slug flow in inclined pipelines described in the next section. This model provided a guide for selecting the operating conditions in an 80-ft. test section. By judiciously combining our experimental and theoretical information, we developed correlations which were successfully validated against field data. We believe that this test of our correlations demonstrates the viability of a fundamental approach to an otherwise complex problem.

A MODEL FOR SLUG FLOW IN INCLINED PIPELINES

In this section we shall describe a model for slug flow which accounts for the pipeline inclination and the gravity forces acting on the liquid slug. The model clearly illustrates the relationship between horizontal and inclined slug flows. Furthermore, it will be recognized that this model is a synthesis of many earlier studies of horizontal and vertical slug flows.

Our discussion begins with a general description of slug flow and the definition of certain key velocities. We then develop an equation for calculating the pressure gradient across a slug unit. Finally, we develop expressions for calculating the important variables in our pressure gradient equation. These variables include the in situ holdup, the holdup in the gas bubble, and a slug-flow friction factor.

Slug flow consists of alternating liquid slugs and gas bubbles. A typical slug unit in an inclined line is shown in Figure 1. In this diagram l_s is the length of the liquid slug in the slug unit having length l_t. The translational velocity of the slug nose is u_T and by continuity this must be the same as the velocity of the gas bubble. The liquid in the slug moves at the particle velocity u_P. This velocity may be expressed in terms of the inlet flow rates by writing, as did Griffith and Wallis (2), a volumetric flow balance around a control volume which encloses the liquid slug and the pipeline inlet. This balance gives

$$u_P = u_L^* + u_G^* = u_{NS} \tag{1}$$

where we have introduced the superficial velocity of the gas and liquid, u_G^* and u_L^*, respectively and u_{NS}, the no-slip velocity. Note that the no-slip velocity is equivalent to the mixture velocity. This derivation shows that the average liquid velocity in the slug [called the particle velocity by Hubbard (3)] is equal to the no-slip or mixture velocity.

In order to calculate the pressure drop along a slug unit, we must write a momentum or force balance for the slug unit. To facilitate the writing of this expression, it is useful to consider the slug at rest. This may be accomplished by imposing a negative velocity u_T on the system. Figure 2

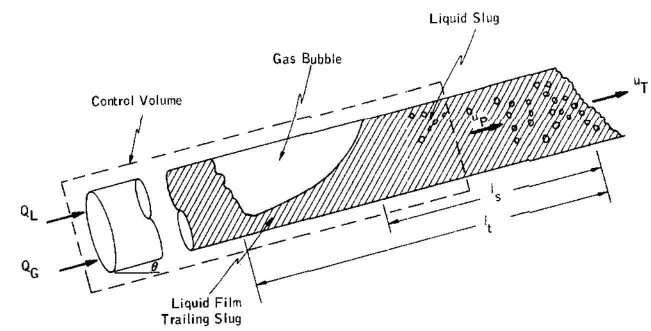

Fig. 1. Slug flow in an inclined pipeline.

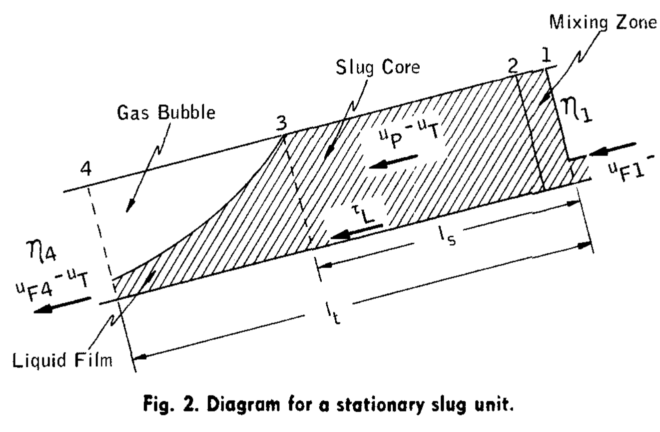

Fig. 2. Diagram for a stationary slug unit.

Correspondence concerning this article should be addressed to Mr. R. H. Bonnecaze. E. J. Greskovich is with Bucknell University, Lewisburg, Pennsylvania 17837.

depicts this situation. In this figure we have idealized the flow by assuming a flat slug nose. Also, note that η is the liquid holdup at any cross section in the slug unit.

A mass balance on the liquid entering and leaving the slug unit gives

$$\eta_4(u_{F4} - u_T) = \eta_1(u_{F1} - u_T) \tag{2}$$

where u_F is the liquid velocity in the film at the point indicated. Because the preceding and succeeding slug units are assumed to be identical in this model, it must be true that $\eta_4 = \eta_1$. Consequently, $u_{F4} = u_{F1} = u_F$. A momentum balance per unit area along the slug unit gives

$$\underbrace{\frac{\rho_L \eta_1}{g_c}(u_{F1} - u_T)^2}_{\text{Momentum flowing in}} - \underbrace{\frac{\rho_L \eta_4}{g_c}(u_{F4} - u_T)^2}_{\text{Momentum flowing out}}$$

$$+ \underbrace{\Delta P_{12}}_{\text{Pressure drop in mixing zone}} + \underbrace{\Delta P_{23}}_{\text{Pressure drop in slug core due to friction and gravity}} + \underbrace{\Delta P_{34}}_{\text{Pressure drop in gas bubble}} + \underbrace{P_1 - P_4}_{\text{Pressure forces}} = 0 \tag{3}$$

Since the liquid film velocities at 1 and 4 are identical, the momentum terms cancel. Therefore

$$\Delta P_{\text{slug}} = P_4 - P_1 = \Delta P_{12} + \Delta P_{23} + \Delta P_{34}$$
$$= \Delta P_{\text{mix}} + \Delta P_{\text{liq}} + \Delta P_{\text{gas}} \tag{4}$$

Equation (4) expresses the pressure drop across a slug unit as the sum of three terms: (1) a pressure drop in the mixing zone, (2) a pressure drop in the slug core which is essentially all liquid, and (3) a pressure drop in the gas bubble. For our purposes we shall assume that the pressure drop in the mixing zone is zero. Although this assumption is not strictly correct, it appears to be useful at this time, since very little information exists for actually computing this contribution. Also we shall set $\Delta P_{\text{gas}} = 0$, because it has been experimentally demonstrated that the pressure drop across the gas bubble is negligible. Therefore the pressure drop across the slug unit is just the pressure drop across the liquid slug.

The pressure drop in the slug core P_{liq} arises primarily from the wall shear forces and gravitational forces acting on the liquid. If we use the Fanning equation to calculate the frictional pressure drop, we have

$$\Delta P_{\text{liq}} = l_s \left(\frac{g}{g_c} \rho_L \sin\theta + \frac{2\rho_L f_L u_{NS}^2}{g_c D} \right) \tag{5}$$

Dividing by l_t and letting $l_s/l_t = \alpha$ be the fraction of the pipeline occupied by liquid slugs, we obtain the pressure gradient for slug flow:

$$\left(\frac{dP}{dx} \right)_{\text{slug}} = \alpha \left[\frac{g}{g_c} \rho_L \sin\theta + \frac{2\rho_L f_L u_{NS}^2}{g_c D} \right] \tag{6}$$

Equation (6) is an important and fundamental equation because it illustrates how the gravitational and frictional forces should be combined for slug flow in arriving at an overall pressure gradient. The Fanning friction factor (f_L) is not necessarily the same as that applicable to flow in a pipeline, because in slug flow the velocity profile of the liquid in the slug core is not likely to be symmetrical as it is in pipeline flow. Because the velocity profile in the slug is difficult to predict, it was necessary to develop a new friction factor correlation from experimental measurements.

In order to use Equation (6) it is necessary to know α, the ratio of slug length to the slug unit length (l_s/l_t). This ratio may be evaluated if we know η, the in situ holdup and η_F, the average liquid holdup in the gas bubble. By equating the total liquid in a slug unit to the liquid in the slug and in the film, we obtain

$$\underbrace{l_s A_P}_{\text{Liquid in the slug}} + \underbrace{(l_t - l_s)\eta_F A_P}_{\text{Liquid in the film}} = \underbrace{l_t A_P \eta}_{\text{Liquid in the slug unit}} \tag{7}$$

where we have assumed that the holdup in the slug is equal to 1. Division of Equation (7) by l_t and rearrangement give

$$\alpha = \frac{l_s}{l_t} = \frac{\eta - \eta_F}{1 - \eta_F} \tag{8}$$

The total holdup for slug flow may be derived from an analysis of the flow dynamics. The film holdup will be discussed in a later section.

EVALUATION OF HOLDUP IN SLUG FLOW

The holdup in slug flow may be related to the slug translational velocity, no-slip velocity, and the flowing volume fraction of liquid. Indeed Griffith and Wallis (2) show that the holdup for slug flow is

$$\eta = 1 - (1 - \lambda)/(u_T/u_{NS}) \tag{9}$$

where we have introduced λ, the flowing volume fraction of liquid.

Equation (9) is a general equation for calculating holdup in slug flow. However, in order to apply this equation successfully we must know the slug translational velocity. From continuity we know that the gas bubble is traveling at the same velocity as the slug. Since this is the case, we shall focus our attention on the gas bubble velocity.

The gas bubble velocity is the sum of two components. The first component arises because of the buoyancy force acting on the gas bubble. We shall call this velocity component the bubble rise velocity, u_{BR}. It is equivalent to the velocity of a large bubble rising in a stagnant column of liquid. The second component arises because the liquid in the slug is not stationary, but instead is moving at the no-slip velocity. Since the gas bubble is traveling relative to the liquid in contact with the nose of the bubble, we must multiply the no-slip velocity by a factor which accounts for the velocity profile in the slug. Adding these two components together gives the following expression for the slug translational velocity:

$$\frac{u_T}{u_{NS}} = C_1 + \frac{\delta u_{BR}}{u_{NS}} \tag{10}$$

In Equation (10) we have introduced the factor δ which indicates the direction in which the buoyancy force is acting. For slug flow in a horizontal pipe, $\delta = 0$ because the buoyancy force does not act in the direction of flow. For uphill slug flow, $\delta = +1$ because the buoyancy force is acting so as to force the gas bubble up the pipe. In downhill slug flow the same phenomenon occurs, but in this case the flow is down the pipe and therefore $\delta = -1$. Equation (10) shows that the slugs may be moving at very low velocities in downhill situations because the two terms in Equation (10) tend to cancel each other. It is interesting to note that this phenomenon has been observed in our experimental tests.

Equation (10) is of the same form as the expressions developed by Griffith and Wallis (2) and by Hubbard

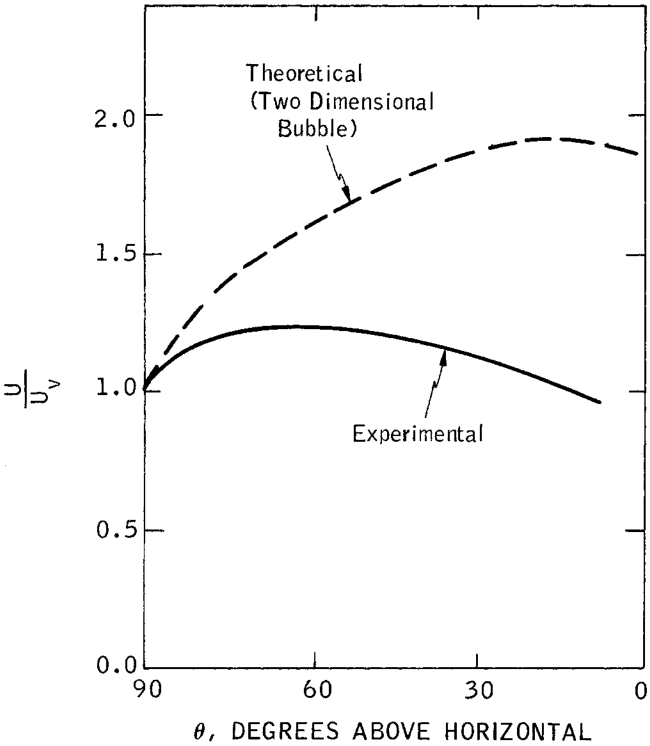

Fig. 3. Bubble rise velocity in inclined tubes.

and Dukler (*3*) for calculating the slug translational velocity in vertical and horizontal slug flows, respectively. Therefore it is interesting to compare the value of $C_1 = 1.20$ obtained from our experimental holdup measurements of slug flow in a test section inclined between −10 and +10 deg. from the horizontal with values of C_1 reported in the literature. Table 1 summarizes the value of C_1 for various pipeline orientations. This tabulation suggests that vertical, horizontal, and inclined slug flows are essentially of the same nature.

The bubble rise velocity u_{BR} for large gas bubbles rising in a vertical tube was derived by Taylor (*7*). Taylor demonstrated that the bubble rise velocity could be calculated from a potential flow model when inertia forces dominated the flow. He concluded that the bubble rise velocity in a stagnant vertical column of liquid is given by

$$u_{BR} = 0.327(1 - \rho_G/\rho_L)\sqrt{gD} \qquad (11)$$

Experimental measurements have shown the constant to be 0.35, which is close to Taylor's value of 0.327.

TABLE 1. VELOCITY PROFILE FACTOR FOR SLUG FLOW

Pipeline orientation	C_1	Source
Vertical	$1.15 - 1.6$ (Diam. = 2 in. − 18 in.) $C_1 = f(N_{Re})$ Avg $C_1 = 1.22$	Griffith (*1*)
	$C_1 = 1.2$	Nicklin (*5*)
Horizontal	$C_1 = 1.25$	Hubbard, Dukler (*3*)
	$C_1 = 1.22$ for $N_{Re} > 10^5$	Hughmark (*4*)
Inclined from −10 to +10 deg. from the horizontal	$C_1 = 1.20$	This work

An interesting phenomenon arises when the pipeline is tilted from the vertical position (*8*). For small inclinations from the vertical the bubble rise velocity increases. The bubble rise velocity continues to increase as the pipe is tilted more and more to the horizontal position until a maximum bubble rise velocity is reached. Increasing the angle of the pipe further results in a decrease in the bubble rise velocity to about the vertical bubble rise velocity when the pipe is about 80 deg. from the vertical. Experimental data beyond 80 deg. is lacking, but our holdup data indicate that the bubble rise velocity is substantial even for 2-deg. inclines above the horizontal. Figure 3 is a schematic representation of these results in which u is the bubble rise velocity in an inclined pipeline and u_v is the velocity at 90 deg. from the horizontal.

In Figure 3 we have superimposed the results from a potential flow model that we have developed for calculating the bubble rise velocity of a two-dimensional bubble rising between two flat plates. In Figure 4*c* we define the coordinates for this model. Note that the bubble is assumed to be at rest and liquid is flowing past the gas bubble. Bernoulli's equation applied to a streamline defining the bubble shape is

$$\frac{P}{\rho_L} - gh + \frac{q^2}{2} = E_L \qquad (12)$$

where $h = y \cos\theta + x \sin\theta$ = vertical distance from the nose of the bubble to a point on the bubble surface and q = liquid velocity on the bubble. For the gas bubble we have

$$\frac{P}{\rho_G} - gh = E_G \qquad (13)$$

Subtracting Equations (12) and (13) and recognizing that at the bubble nose we have a stagnation point ($q = 0$), we find that

$$q^2 = 2(1 - \rho_G/\rho_L)gh = 2(1 - \rho_G/\rho_L)\, g(y \cos\theta + x \sin\theta) \qquad (14)$$

Equation (14) shows that the liquid velocity for a point on the gas bubble is proportional to its vertical distance from the nose of the bubble. In Figure 4 we illustrate how this distance (h) changes as the bubble is rotated from the vertical position. For small angles from the vertical (Figure 4*b*) we notice that the vertical distance from the bubble nose to point P is actually greater than that for the vertical case (Figure 4*a*). Since $h' > h''$ we have, from Equation (14), that the velocity of the liquid at point P would increase as the bubble is tilted from the vertical. However, when the gas bubble is nearly horizontal (Figure 4*c*) we notice that $h < h''$, and hence the liquid velocity at point P is less than in the vertical case. Notice that this analysis provides an explanation of the observation that the bubbble rise velocity first increases and then decreases as the pipeline is tilted from the vertical position.

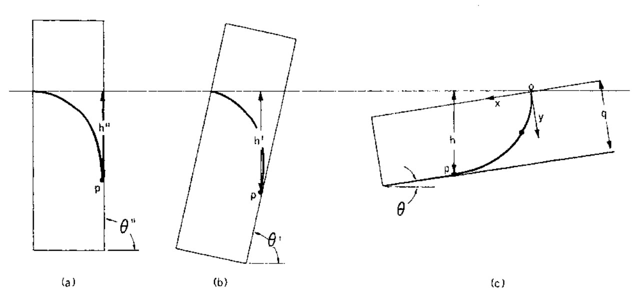

Fig. 4. Effect of rotation on a large gas bubble rising in a tube.

The above explanation may be quantified by applying to our problem the same procedure that Taylor (8) used in arriving at Equation (11). In order to calculate the liquid velocity q, it is necessary to know the velocity distribution around the bubble. This may be obtained by calculating the velocity potential for the fluid flowing between the flat plates. From this we can determine the stream function for the flow. These equations are combined with the Bernoulli equation (in a manner exactly analogous to Taylor) to give the following equation for the bubble rise velocity as a function of inclination:

$$\frac{u}{u_v} = \frac{\sqrt{\cos\theta + \frac{2}{\pi}\ln\left(\frac{\pi}{2}\right)\sin\theta}}{\sqrt{\frac{2}{\pi}(\ln \pi/2)}}$$

$$= \sqrt{3.43\cos\theta + \sin\theta} \quad (15)$$

Equation (15) is shown in Figure 3. Note that the observed trends are predicted by the model. Of course, we would not expect quantitative agreement because the flow geometry analyzed was not cylindrical. In addition, our analysis predicts a nonzero bubble rise velocity when the tube is in the horizontal position. This is clearly impossible because the buoyancy forces are absent when the flow is horizontal. The result apparently arises because our model assumes a constant bubble shape which is not the case. Nevertheless, it is interesting to note that our model does predict a rise in bubble velocity as the line is inclined from the vertical. Furthermore, the model suggests that the bubble rise velocity remains significant even at small inclinations above the horizontal. Indeed, the holdup measurements in our 80 ft. 1½ in. I.D. test section inclined from −10 to +10 deg. were successfully correlated by assuming that $u_{BR} = 0.35\,(1 - \rho_G/\rho_L)\sqrt{gD}$, which corresponds to a rather large bubble rise velocity for small inclinations from the horizontal but which is certainly consistent with our potential flow model. Incorporating this value of u_{BR} and $C_1 = 1.20$ into Equation (10) and introducing the result into Equation (9), we get the following equation for slug-flow holdup in an inclined pipeline:

$$\eta = 1 - \frac{(1-\lambda)}{1.20 + 0.35(1 - \rho_G/\rho_L)/\delta\sqrt{N_{Fr}}} \quad (16)$$

where $\delta = 0$ for horizontal flow, $= +1$ for uphill flow, $= -1$ for downhill flow.

In order to compare Equation (16) against the experimental data gathered in our laboratory, we shall define the average deviation and the standard deviation as

$$\%\ \text{deviation} = 100\bar{d} = \frac{1}{N}\sum_{i=1}^{N} d_i \times 100 \quad (17)$$

and

$$\%\ \text{standard deviation} = \sqrt{\sum_{i=1}^{N}\frac{(d_i - \bar{d})^2}{N-1}} \times 100 \quad (18)$$

where

$$d_i = \frac{\eta_{\text{calc}} - \eta_{\text{measured}}}{\eta_{\text{measured}}}$$

Table 2 compares the average deviation against the measured holdup for slug flow in our test section inclined at the indicated angles. Equation (16) is also in agreement with the data of Odishariya et al. (6).

TABLE 2. COMPARISON OF PREDICTED HOLDUP VERSUS EXPERIMENTAL HOLDUP FOR SLUG FLOW IN AN INCLINED PIPELINE

Slope, deg	No. of points	% deviation	% standard deviation
+10	48	11.7	14.9
+6	44	11.0	11.1
+2	36	23.0	20.1
0	18	−6.1	4.8
−2	4	−5.5	19.1
−6	3	−9.5	9.6
−10	1	−6.0	—

EVALUATION OF FILM HOLDUP

In order to calculate α [Equation (8)] we need, in addition to the total holdup, a knowledge of η_F, the liquid holdup in the film trailing the slug. In this section we present a technique for calculating η_F for uphill, downhill, and horizontal flows.

Since the average velocity across the gas bubble is u_{NS} and the velocity of the gas within the gas bubble is u_T, the slug translational velocity, we can write

$$u_{NS} = (1 - \eta_F)u_T + u_F\eta_F$$

where u_F = velocity of the liquid in the film. This equation can be solved with a force balance on the liquid to obtain an expression for η_F. However, for horizontal flow this force balance is difficult to express mathematically. Consequently, we shall assume that the velocity in the film is zero, so that in horizontal flow

$$\eta_F = 1 - \frac{u_{NS}}{u_T} = 1 - \frac{1}{C_1} = \frac{C_1 - 1}{C_1} \quad (19)$$

Substitution of this value of η_F into Equation (8) and the holdup expression for horizontal slug flow gives

$$\alpha = \lambda \quad (20)$$

Equation (20) shows that the ratio of slug length (l_s) to slug unit length (l_t) is approximately equal to the flowing volume fraction of liquid.

For uphill and downhill slug flows, we assume that within the gas bubble, a balance between the gravitational and frictional forces is achieved for the liquid. The resulting force balance is identical to that derived for downhill stratified flow assuming open-channel flow. If this equation is solved simultaneously with the volume balance across the gas bubble, we obtain the result

$$\eta^3{}_F/\epsilon_F = \frac{2f_L\,Fr\,\lambda^2{}_F}{|\sin\theta|} \quad (21a)$$

$$\lambda_F = \delta\left[\frac{u_T}{u_{NS}}(1 - \eta_F) - 1\right] \quad (21b)$$

where

$$\eta_F = \frac{1}{\pi}\left[\cos^{-1}(1-y) - (1-y)y^{1/2}(2-y)^{1/2}\right] \quad (21c)$$

$$\epsilon_F = \frac{1}{\pi}\cos^{-1}(1-y) \quad (21d)$$

$$y = 2h/d \quad (21e)$$

In these equations h is the depth of liquid in the pipe when the flow is stratified, θ is the slope of the pipeline, and δ is included to account for the fact that the liquid flow directions are different in uphill and downhill flows. Substitution of this value of η_F into the expression for α completes the derivation of the equations for calculating pressure drop in inclined flow.

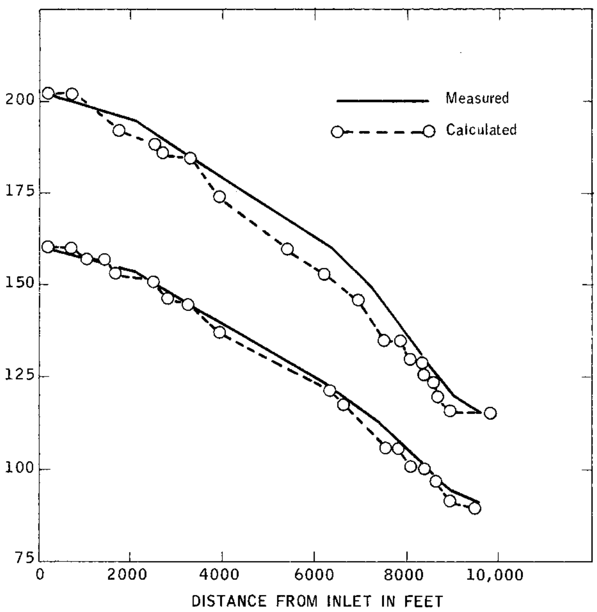

Fig. 5. Comparison of two-phase flow pressure drop model with field data; 6-in. line.

COMPARISON OF MEASURED AND PREDICTED PRESSURE DROP FOR SLUG FLOW

The above development has shown how a simple mechanistic model was used to develop a set of equations for calculating holdup and pressure drop in slug flow. As mentioned previously, it is necessary to develop a slug flow friction factor to correctly compensate for the velocity profile in the slug core. From Equations (6), (8), (16), and (21) and our experimental data we were able to correlate the pressure drop measurements with the following friction factor:

$$f_{SF} = 0.0048 + 3980./N_{Re}^{1.285} \quad (22)$$

In Table 3 we compare our measured pressure drop against the calculated pressure drop using Equation (22). The percent deviation and the percent standard deviation are defined in Equations (17) and (18), and where

$$d_i = \frac{\Delta P_{\text{calc}} - \Delta P_{\text{measured}}}{\Delta P_{\text{measured}}}$$

As a final check of our model, we have compared our predicted pressure drop against actual field data from an oil-gas system. Pressure profiles were obtained over a 10,000-ft. section of a 6-in. line. Figure 5 shows that the maximum deviation is about 5%. As stated above, stratified flow dominates in downhill lines and this was taken into consideration for the pressure drop calculations.

CONCLUSIONS

Based on the above summary of our two-phase flow study, we can list our accomplishments as follows:

Derivation and verification of an in situ holdup model for inclined slug flow.

Formulation of a two-phase slug-flow friction factor.

Derivation of a two-phase slug-flow pressure drop for inclined pipelines verified by field measurements.

ACKNOWLEDGMENT

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TABLE 3. COMPARISON OF PREDICTED PRESSURE DROP VERSUS MEASURED PRESSURE DROP FOR SLUG FLOW IN INCLINED PIPELINES

Slope, deg	No. of points	% deviation	% standard deviation
+10	48	−1.9	9.0
+6	44	5.5	13.7
+2	36	12.2	12.3
0	18	−0.2	5.1
−2	4	221.0*	94.3
−6	3	22.7	25.0
−10	1	21.8	—

* The pressure drops were so low that small absolute deviations resulted in rather high percentages.

NOTATION

A_P = cross-sectional area of pipe, ft.
D = pipe diameter, ft.
f_L = friction factor
f_{SF} = slug flow friction factor
g = acceleration of gravity, ft./sec.
g_c = conversion factor, 32.174 lb.$_m$-ft./lb.$_f$-sec.^{2}
h = height of liquid in pipe, ft.
N_{Fr} = Froude number = u^2_{NS}/gD
N_{Re} = Reynolds number = $Du_{NS}\rho_L/\mu_L$
P = pressure, lb./sq.in.abs.
ΔP = pressure drop, lb./sq.in.
Q_L = volumetric flow rate of liquid, cu.ft./sec.
Q_g = volumetric flow rate of gas, cu.ft./sec.
u_{NS} = no-slip velocity = $(Q_G + Q_L)/A_P$, ft./sec.
u_T = translational velocity of liquid slug, ft./sec.
u_{BR} = bubble rise velocity, ft./sec.
u_G^*, u_L^* = superficial velocities, ft./sec.
V_L = volume of liquid, cu.ft.
V_G = volume of gas, cu.ft.
y = dimensionless distance $(2h/D)$

Greek Letters

α = ratio of liquid slug to slug unit $(\eta - \eta_F)/(1 - \eta_F)$
ϵ_F = geometric factor for film holdup
η = in situ holdup
η_F = film holdup
θ = angle of inclination from horizontal (radians, unless specified otherwise)
λ = liquid volume fraction entering pipeline $Q_L/(Q_L + Q_G)$
μ_L = liquid viscosity, lb./(ft.)(sec.)
ρ_L = liquid density, lb./cu.ft.

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